MU123

TMA 04 2019B

Covers Units 10, 11 and 12 **Cut-off date 13 August 2019**

Submission instructions

You will find instructions for completing TMAs in the 'Assessment' area of the MU123 website. Please read these instructions before beginning work on this TMA.

Reviewing your tutor's comments on your previous TMA will help you as you work on this one.

Special instructions

Remember that you need to explain your reasoning and communicate your ideas clearly, as described in Subsection 5.3 of Unit 1. This includes:

- explaining your mathematics in the context of the question
- the correct use of notation and units
- appropriate rounding.

Your score out of 5 for good mathematical communication (GMC) will be recorded against Question 8. You do not have to submit any work for Question 8.

TMA 04   **Cut-off date** 13 August 2019

Question 1 –   20 marks

This question is based on your work on MU123 up to and including Unit 10.

Marcus competes in the shot put for his local track and field sports club. When he throws the shot, its trajectory after he releases it can be modelled by the quadratic equation

$$y = -\frac{x^2}{10} + \frac{9x}{10} + \frac{11}{5},$$

where y represents the height in metres of the shot above the ground, and x represents the horizontal distance in metres of the shot from the position where it is released by Marcus. Assume that the surface of the field is horizontal.

(a) The graph of $y = -\dfrac{x^2}{10} + \dfrac{9x}{10} + \dfrac{11}{5}$ is a parabola.

   (i)   Is the parabola u-shaped or n-shaped? How can you tell this from the equation?   [1]

   (ii) Use algebra to find the x-intercepts.   [4]

   (iii) Explain why the y-intercept is $\frac{11}{5}$.   [2]

   (iv) Find the equation of the axis of symmetry, explaining your method. Use this information to find the coordinates of the vertex, rounding your answers where necessary to one decimal place.   [4]

   (v)   Provide a sketch of the graph of the parabola, either by hand or by using Graphplotter.   [3]

         You should refer to the graph-sketching strategy box in Section 2.4 of Unit 10 for information on how to sketch and label a graph correctly.

(b) In this part of the question, you are asked to consider the trajectory of the shot modelled by the equation $y = -\dfrac{x^2}{10} + \dfrac{9x}{10} + \dfrac{11}{5}$ in conjunction with the results that you found in part (a).

   (i)   Find the height of the shot when it is 3 metres horizontally from the position where Marcus releases it.   [1]

   (ii) Use your answer to part (a)(iv) to find the maximum height reached by the shot.   [2]

   (iii) What does the y-intercept represent in the context of this model? [1]

   (iv) How far will the shot be horizontally from the position where Marcus releases it when it first lands on the field? Explain your answer.   [2]

Question 2 – 13 marks

This question is based on your work on MU123 up to and including Unit 10.

(a) Use the quadratic formula to solve the equation

$$5t^2 - 14t + 6 = 0.$$

Give your answers correct to one decimal place. [3]

(b) This part of the question concerns the quadratic equation

$$5x^2 - 11x + 8 = 0.$$

   (i) Find the discriminant of the quadratic expression $5x^2 - 11x + 8$. [2]

   (ii) What does this tell you about the number of solutions of the equation? Explain your answer briefly. [2]

   (iii) What does this tell you about the graph of $y = 5x^2 - 11x + 8$? [1]

(c) (i) Write the quadratic expression $x^2 - 24x - 12$ in completed-square form. [2]

   (ii) Use the completed-square form from part (c)(i) to solve the equation $x^2 - 24x - 12 = 0$, leaving your answer in exact (surd) form, simplified as far as possible. [2]

   (iii) Use the completed-square form from part (c)(i) to write down the vertex of the parabola $y = x^2 - 24x - 12$. [1]

Question 3 – 18 marks

This question is based on your work on MU123 up to and including Unit 11.

A company wished to know if the training programme that they developed for a particular task was effective. 20 employees were timed performing the task before and after the training. The times were recorded and are given in Table 1.

Table 1   Time spent performing the task (in minutes)

Before training	After training
27	24
28	23
22	20
26	24
21	21
31	24
29	24
27	23
29	22
29	25
28	23
28	24
28	25
27	22
29	23
28	22
26	23
30	24
26	23
25	22

(a) Enter these data into two lists in Dataplotter.

To check that you have entered the values correctly, the mean number of minutes that it took to perform the task before training is 27.2 minutes, and the mean number of minutes it took to perform the task after training is 23.1 minutes.

Create boxplots for the two datasets, either using Dataplotter or by hand. Include either a printout of your boxplots or your complete hand-drawn boxplots with your answer to this question.

*Remember to include all relevant information on your boxplots as set out in Subsection 1.2 of Unit 11. Using Dataplotter, it is sufficient to include the key values in the list to the right of the boxplot, rather than on the boxplot itself. If you draw boxplots by hand, then you should use squared paper and a common axis for both plots to make it easy to compare the boxplots. Remember that the mean and standard deviation are **not** part of a boxplot.* [7]

(b) A boxplot gives you a visual representation of the average value using the **median**, and also tells you how the data are spread out based on the size of the box and the lengths of the whiskers.

    (i)   How do the average times compare for performing the task before training and after training? Use your **boxplots** from part (a) to explain your answer.   [2]

    (ii) Are the data more spread out for performing the task before training or after training? Use your **boxplots** from part (a) to explain your answer.   [2]

(c) Use the boxplot for before training to say whether the data are symmetrical or skewed. If the data are skewed, then state whether they are skewed to the left or skewed to the right, explaining your reasoning briefly.   [2]

(d) Create a histogram for each of the datasets, using a start value of 20 and an interval of 1. Include either a printout of your histograms or a sketch drawn by hand with your answer to this question.   [3]

If you draw histograms by hand, then you should use squared paper and the same axis scale for both histograms to make it easy to compare them.

(e) Comment on one aspect of the time spent performing the task that can be seen more easily on the histograms than on the boxplots.   [2]

Question 4 – 19 marks

This question is based on your work on MU123 up to and including Unit 12.

(a) Find the length of the side marked x in the triangle in Figure 1, giving your answer correct to the nearest cm.   [3]
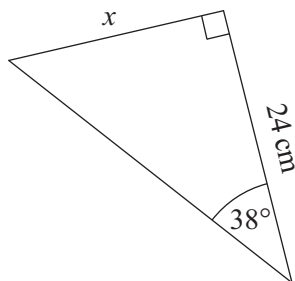

Figure 1

(b) Triangle PQR has a right angle at R. The length of side PQ is 24.5 cm, and the length of side QR is 15 cm. Draw triangle PQR, and find $\angle RPQ$, giving your answer correct to the nearest degree.   [3]

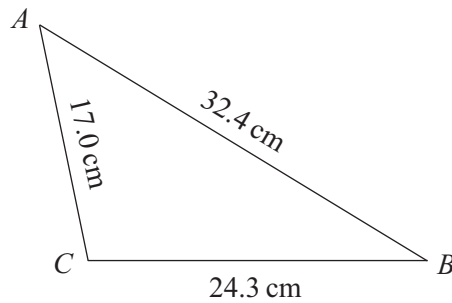(c) (i) Find the angle ABC in the triangle in Figure 2, giving your answer correct to the nearest degree. [5]

Figure 2

(ii) Find the area of the triangle ABC in Figure 2, giving your answer correct to the nearest square cm. [3]

(d) (i) Convert $72°$ to radians, leaving your answer in terms of π. [2]

(ii) Use your answer from part (d)(i) to find the area of a sector of a circle of radius 8.2 cm and angle $72°$, giving your answer correct to two significant figures. [3]

Question 5 – 10 marks

This question is based on your work on MU123 up to and including Unit 12.

You should use trigonometry, not scale drawings, to find your answers. Give all answers correct to two significant figures. Remember to use full versions of earlier answers to avoid rounding errors.

Tom and his two friends Joe and Colm are playing catch with a beach ball at the seaside. Colm is standing at a point C, 24 metres due south of Joe, who is standing at point J. Tom is standing at a point T, $52°$ west of north of Colm and 27 metres from point C. (You may assume that all distances are flat and in a straight line.)

(a) Draw a diagram showing the points T, J and C (triangle TJC), marking the internal angle and the lengths that you are given. [2]

(b) Find the distance between Tom and Joe, that is, the length of the side TJ. [3]

(c) When Tom throws the beach ball, the wind catches it so that it lands on the ground between Joe and Colm (on the line JC) due east of Tom (T). Add a line to your diagram that shows the shortest distance from where Tom is standing (point T) to where the ball lands at point B (on the line JC). What angle does the line TB make with the line JC? [2]

(d) How far is Tom from the ball now? That is, find the length of TB. [3]

Question 6 – 10 marks

This question is based on your work on MU123 up to and including Unit 12.

In this question, you are asked to comment on a student's incorrect attempt at answering the question detailed below.

(a) Write out your own solution to the question, explaining your working. [4]

(b) There are two places in the student's attempt where a mistake has been made. Identify these mistakes and explain, as if directly to the student, why, for each mistake, their working is incorrect. [6]

The Question
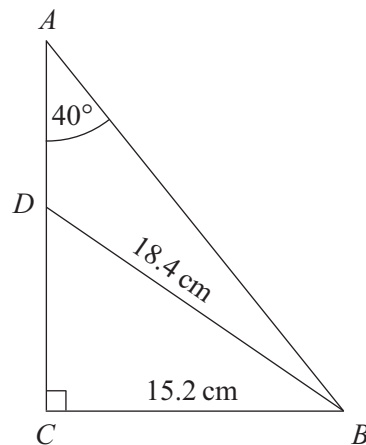
Triangle ABC is a right-angled triangle. The length of side BC is 15.2 cm. $\angle CAB = 40°$ and $\angle BCA = 90°$. D is a point on side CA, and BD has length 18.4 cm.

(i) Find $\angle CDB$ and hence $\angle ADB$, giving your answers to two significant figures.

(ii) Find the length of DC, giving your answer to two significant figures.

(iii) Use your answer to part (i) to find the length of AB, giving your answer to two significant figures.

The student's incorrect attempt

(i) In triangle DBC, the known sides are the hypotenuse and the side opposite to $\angle CDB$, so the ratio needed is sine.

$$\sin (\angle CDB) = \frac{15.2}{18.4}$$

$$\angle CDB = \sin^{-1} \left(\frac{15.2}{18.4} \right) = 55.6 \dots°$$

So $\angle CDB$ is 56° (to 2 s.f.)

ACD is a straight angle so is 180°

Therefore $\angle ADB = 180° - 55.6\dots° = 124.3\dots°$

So $\angle ADB$ is 120° (to 2 s.f.)

(ii) Triangle BCD is a right-angled triangle so
using Pythagoras' Theorem

$DB^2 = CB^2 + DC^2$

$DC^2 = DB^2 - CB^2$

$DC = DB - CB$

$DC = 18.4 - 15.2 = 3.2$

The length of DC is 3.2 cm (to 2 s.f.)

(iii) Using the sine rule in triangle ABD

$$\frac{AB}{\sin (\angle ADB)} = \frac{DB}{\sin (\angle DAB)}$$

$$\frac{AB}{\sin (120°)} = \frac{18.4}{\sin (40°)}$$

$$AB = \frac{18.4 \times \sin (120°)}{\sin (40°)} = 24.7 \dots$$

The length of AB is 25 cm (to 2 s.f.)

Question 7 – 5 marks

Two of the themes that you have met in MU123 are:

- working with data
- algebraic skills.

In this question you are asked to think about your progress so far with **one** of these two themes.

(a) Choose one of the two themes mentioned above (either working with data or algebraic skills). If you plan to continue studying mathematics, science or engineering, then we recommend that you choose algebraic skills because this will be an important aspect of later work. Otherwise, choose either theme.

Write down the theme that you have chosen. Write down one topic in your chosen theme that you can work with confidently, and one topic in your chosen theme that you find more challenging. [2]

(If you did not find any of the work in your chosen theme challenging, then pick two topics that you can work with confidently.)

(b) Describe two steps that you could take to help you to work more confidently with the topic from part (a) that you find challenging. [2]

(If you did not find any of the work in your chosen theme challenging, then explain why.)

(c) Give one example from this TMA of your chosen theme where you were able to check your answer. How did you check it? [1]

Question 8 – 5 marks

A score out of 5 marks for good mathematical communication over the entire TMA will be recorded under Question 8. [5]